The typical text used for this course and excellent reference is entitled Mechanics of Materials by Timothy A. Philpot. All examples and solutions below are taken from this text and the instructor solutions.

MIE 211 - *Strength [and Mechanics] of Materials*

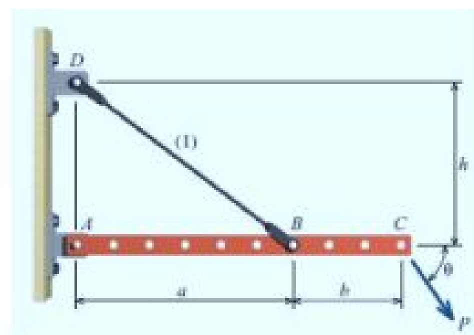
Strength of Materials (alternatively known as Mechanics of Materials), is the basis of predicting the behavior of materials when different types of forces or loading is applied. On an undergraduate level, most focus is maintained on the material behavior of standard metals, such as aluminum and steel. The main topics covered in Strength of Materials, and locations of useful links in understanding how to solve the various problems, are:

- Introduction to Stress and Strain (https://bit.ly/30Qm3fS)
  - This video provides a basic introduction to stress, strain, and their relationships to material deformation and material constants, such as the Modulus of Elasticity (Young's Modulus) and Poisson's Ratio.

Example Problem:

Problem 1. (5 Points) ABC is a rigid bar. At point A, a single-shear pin connection supports the bar. At points B and D, double-shear pin connections support a rod (1). The diameter of rod (1) is 1/4 inch. The diameters of all pins are 1/4 inch. (**FBD required**)

Assume a = 2 feet, b = 1 feet, h = 1.5 feet, P = 202 lb, and $\theta = 55°$.
Determine the following:
(a) Normal stress in Rod (1).
(b) Shear Stress in Pin B.
(c) Shear Stress in Pin A.



No Solution Provided

- Basics of Stress Limits (https://bit.ly/30Nc1w4)
  - This video provides an introduction to various aspects of material behavior prediction, including ultimate tensile strength, yield strength, and also gives you a starting basis for completing structural design.

Example Problem

P6.74 The torsional assembly of Figure P6.73/74 consists of a cold-rolled stainless steel tube connected to a solid cold-rolled brass segment at flange *C*. The assembly is securely fastened to rigid supports at *A* and *D*. Stainless steel tubes (1) and (2) have an outside diameter of 3.50 in., a wall thickness of 0.120 in., a shear modulus of G = 12,500 ksi, and an allowable shear stress of 30 ksi. The solid brass segment (3) has a diameter of 2.00 in., a shear modulus of G = 5,600 ksi, and an allowable shear stress of 18 ksi. Determine the maximum permissible magnitude for the concentrated torque T_B.

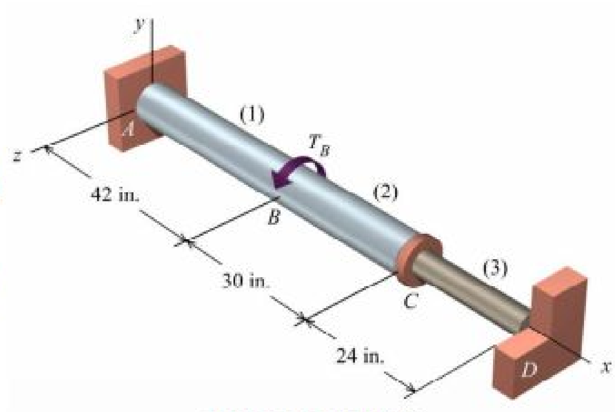


FIGURE P6.73/74

Solution

Solution

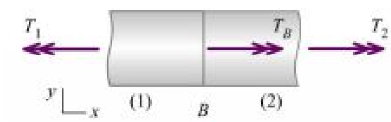
Section Properties: The polar moments of inertia for the stainless steel tube and the solid brass segment are:

$$J_1 = \frac{\pi}{32}\left[(3.500\text{ in.})^4 - (3.260\text{ in.})^4\right] = 3.643915\text{ in.}^4 = J_2$$

$$J_3 = \frac{\pi}{32}(2.0\text{ in.})^4 = 1.570796\text{ in.}^4$$

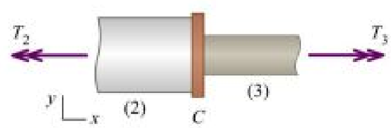
Equilibrium: Consider a free-body diagram cut around joint *B*, where the external torque T_B is applied:

$$\Sigma M_x = -T_1 + T_2 + T_B = 0 \qquad \text{(a)}$$



and also consider a FBD cut around joint *C*. Although there is not an external torque applied at joint *C*, the section properties of the torsion structure change at *C*.

$$\Sigma M_x = -T_2 + T_3 = 0 \qquad \therefore T_2 = T_3 \qquad \text{(b)}$$



Geometry-of-Deformation Relationship: Since the two ends of the torsion structure are securely attached to fixed supports at *A* and *D*, the sum of the angles of twist in the three shafts must equal zero:

$$\phi_1 + \phi_2 + \phi_3 = 0 \qquad \text{(c)}$$

Torque-Twist Relationships:

$$\phi_1 = \frac{T_1 L_1}{J_1 G_1} \qquad \phi_2 = \frac{T_2 L_2}{J_2 G_2} \qquad \phi_3 = \frac{T_3 L_3}{J_3 G_3} \qquad \text{(d)}$$

Compatibility Equation: Substitute the torque-twist relationships [Eqs. (d)] into the Geometry-of-deformation relationship [Eq. (c)] to obtain the compatibility equation:

$$\frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} + \frac{T_3 L_3}{J_3 G_3} = 0 \qquad \text{(e)}$$

Solve the Equations: Since the problem is expressed in terms of allowable stresses, it is convenient to rewrite Eq. (e) in terms of stresses. In general, the elastic torsion formula can be rearranged as:

$$\tau = \frac{Tc}{J} \qquad \therefore \frac{T}{J} = \frac{\tau}{c}$$

which allows Eq. (e) to be rewritten as:

$$\frac{\tau_1 L_1}{G_1 c_1} + \frac{\tau_2 L_2}{G_2 c_2} + \frac{\tau_3 L_3}{G_3 c_3} = 0 \qquad \text{(f)}$$

Also using the elastic torsion formula, Eq. (b) can be expressed in terms of stress:

$$\frac{\tau_2 J_2}{c_2} = \frac{\tau_3 J_3}{c_3} \qquad \therefore \frac{\tau_3}{c_3} = \frac{\tau_2 J_2}{c_2 J_3} \tag{g}$$

Substitute Eq. (g) into Eq. (f) to obtain

$$\frac{\tau_1 L_1}{G_1 c_1} + \frac{\tau_2 L_2}{G_2 c_2} + \frac{L_3}{G_3}\frac{\tau_2 J_2}{c_2 J_3} = 0$$

Simplify:

$$\tau_1 \frac{L_1}{G_1 c_1} = -\tau_2 \left[\frac{L_2}{G_2 c_2} + \frac{L_3}{G_3}\frac{J_2}{c_2 J_3}\right] \qquad \therefore \tau_1 = -\tau_2 \left[\frac{\dfrac{L_2}{G_2 c_2} + \dfrac{L_3}{G_3 c_2}\dfrac{J_2}{J_3}}{\dfrac{L_1}{G_1 c_1}}\right]$$

Substitute values:

$$\tau_1 = -\tau_2 \left[\frac{\dfrac{30 \text{ in.}}{(12{,}500 \text{ ksi})(3.50 \text{ in./2})} + \dfrac{24 \text{ in.}}{(5{,}600 \text{ ksi})(3.50 \text{ in./2})}\left(\dfrac{3.643915 \text{ in.}^4}{1.570796 \text{ in.}^4}\right)}{\dfrac{42 \text{ in.}}{(12{,}500 \text{ ksi})(3.50 \text{ in./2})}}\right]$$

$$= -3.6732\,\tau_2$$

This calculation demonstrates that the shear stress in segment (1) of the stainless steel tube is much larger than the shear stress in segment (2). If the shear stress magnitude in segment (1) is 30 ksi, then the shear stress magnitude in segment (2) will be:

$$\tau_2 = \left|-\frac{\tau_1}{3.6732}\right| = 8.1673 \text{ ksi} \tag{h}$$

Next, we need to check the corresponding shear stress in brass shaft (3). From Eq. (g):

$$\tau_3 = \tau_2 \frac{c_3}{c_2}\frac{J_2}{J_3}$$

Substitute the magnitude obtained for τ_2 in Eq. (h) into this expression and calculate the corresponding shear stress magnitude in shaft (3):

$$\tau_3 = (8.1673 \text{ ksi})\left(\frac{2.00 \text{ in./2}}{3.50 \text{ in./2}}\right)\left(\frac{3.643915 \text{ in.}^4}{1.570796 \text{ in.}^4}\right) = 10.8265 \text{ ksi} \leq 18 \text{ ksi} \qquad \textbf{O.K.}$$

Now that the maximum shear stresses in the three shaft segments are known, the torques in each component can be computed:

$$T_1 = \frac{\tau_1 J_1}{c_1} = \frac{(30 \text{ ksi})(3.643915 \text{ in.}^4)}{3.50 \text{ in./2}} = 62.4671 \text{ kip-in.}$$

$$T_2 = \frac{\tau_2 J_2}{c_2} = \frac{(-8.1673 \text{ ksi})(3.643915 \text{ in.}^4)}{3.50 \text{ in./2}} = -17.0062 \text{ kip-in.}$$

$$T_3 = \frac{\tau_3 J_3}{c_3} = \frac{(-10.8265 \text{ ksi})(1.570796 \text{ in.}^4)}{2.00 \text{ in./2}} = -17.0062 \text{ kip-in.}$$

Maximum Permissible Torque T_B: From Eq. (a), the torque T_B acting on the assembly must not exceed:

$$T_{B,max} = T_1 - T_2 = 62.4671 \text{ kip-in.} - (-17.062 \text{ kip-in.}) = 79.4733 \text{ kip-in.} = \boxed{79.5 \text{ kip-in.}}$$ **Ans.**

❖ Beams (https://bit.ly/2LNowDy)

➢ This video provides an overview on solving statically determinate beams and determining the forces on each joint or at different locations.

Example Problem

P8.22 A flanged wooden shape is used to support the loads shown on the beam in Figure P8.22*a*. The dimensions of the shape are shown in Figure P8.22*b*. Consider the entire 18-ft length of the beam and determine:
(a) the maximum tension bending stress at any location along the beam, and
(b) the maximum compression bending stress at any location along the beam.

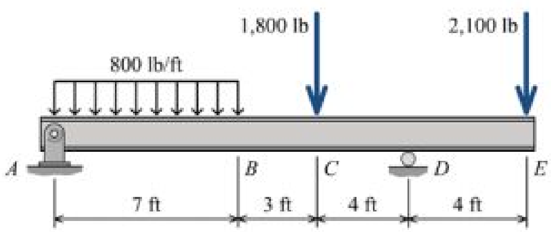


FIGURE P8.22*a*

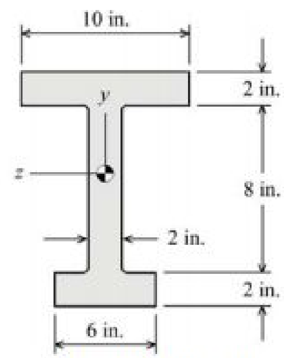


FIGURE P8.22*b*

Solution

Centroid location in *y* direction:

Shape	Area A_i (in.^{2})	y_i (from bottom) (in.)	$y_i A_i$ (in.^{3})
top flange	20.0	11.0	220.0
web	16.0	6.0	96.0
bottom flange	12.0	1.0	12.0
	48.0 in.^{2}		328.0 in.^{3}

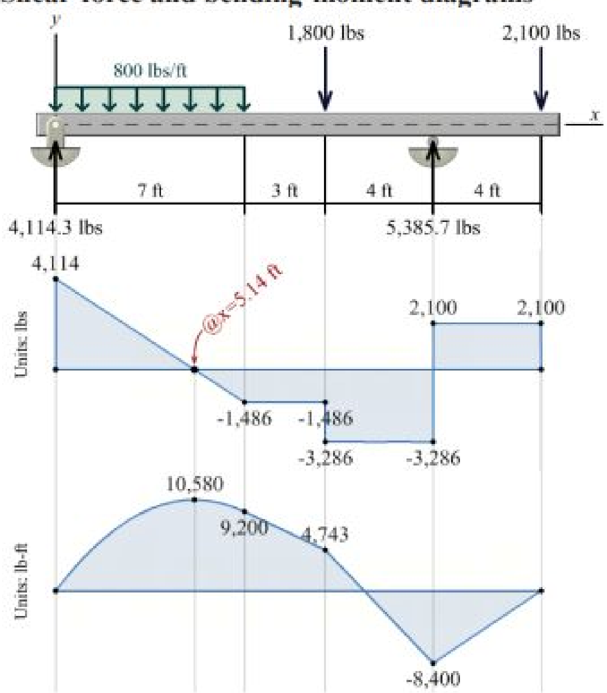
$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{328.0 \text{ in.}^3}{48.0 \text{ in.}^2} = 6.8333 \text{ in.} \quad \text{(from bottom of shape to centroid)}$$

$$= 5.1667 \text{ in.} \quad \text{(from top of shape to centroid)}$$

Moment of inertia about the *z* axis:

Shape	I_C (in.^{4})	$d = y_i - \bar{y}$ (in.)	d^2A (in.^{4})	$I_C + d^2A$ (in.^{4})
top flange	6.667	4.167	347.222	353.889
web	85.333	–0.833	11.111	96.444
bottom flange	4.000	–5.833	408.333	412.333
	Moment of inertia about the *z* axis (in.^{4}) =			862.667

Shear-force and bending-moment diagrams



Maximum bending moments

positive $M = 10{,}580$ lb-ft

negative $M = -8{,}400$ lb-ft

Bending stresses at max positive moment

$$\sigma_x = -\frac{(10{,}580 \text{ lb-ft})(5.1667 \text{ in.})(12 \text{ in./ft})}{862.667 \text{ in.}^4}$$

$$= 760.4 \text{ psi (C)}$$

$$\sigma_x = -\frac{(10{,}580 \text{ lb-ft})(-6.8333 \text{ in.})(12 \text{ in./ft})}{862.667 \text{ in.}^4}$$

$$= 1{,}005.6 \text{ psi (T)}$$

Bending stresses at max negative moment

$$\sigma_x = -\frac{(-8{,}400 \text{ lb-ft})(5.1667 \text{ in.})(12 \text{ in./ft})}{862.667 \text{ in.}^4}$$

$$= 603.7 \text{ psi (T)}$$

$$\sigma_x = -\frac{(-8{,}400 \text{ lb-ft})(-6.8333 \text{ in.})(12 \text{ in./ft})}{862.667 \text{ in.}^4}$$

$$= 798.5 \text{ psi (C)}$$

(a) Maximum tension bending stress = 1,006 psi (T) **Ans.**

(b) Maximum compression bending stress = 799 psi (C) **Ans.**

Example Problem

P9.12 A 5-m long simply supported timber beam carries two concentrated loads, as shown in Figure P9.12*a*. The cross-sectional dimensions of the beam are shown in Figure P9.12*b*.
(a) At section *a–a*, determine the magnitude of the shear stress in the beam at point *H*.
(b) At section *a–a*, determine the magnitude of the shear stress in the beam at point *K*.
(c) Determine the maximum horizontal shear stress that occurs in the beam at any location within the 5-m span length.
(d) Determine the maximum compression bending stress that occurs in the beam at any location within the 5-m span length.

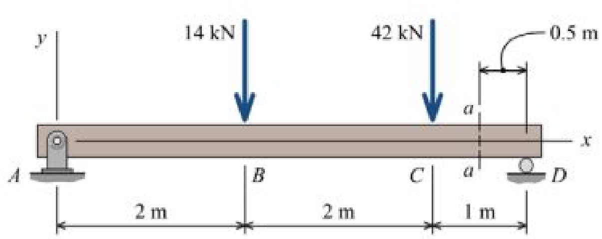


FIGURE P9.12*a* Simply supported timber beam

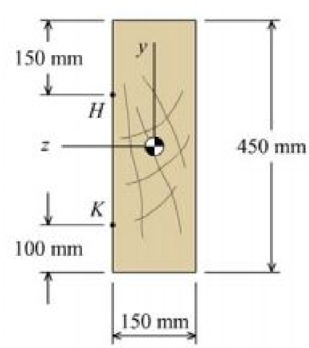
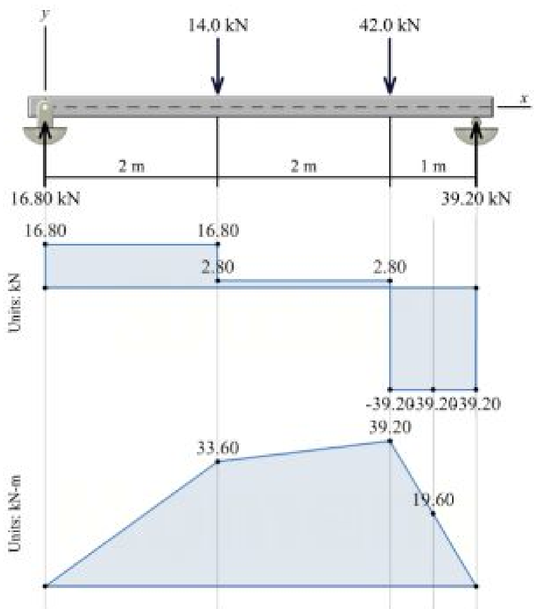


FIGURE P9.12*b* Cross-sectional dimensions

Solution

Section properties:

$$I = \frac{(150\text{ mm})(450\text{ mm})^3}{12} = 1{,}139.1 \times 10^6\text{ mm}^4 \qquad t = 150\text{ mm}$$



(a) Shear stress magnitude at *H*:

$$Q = (150\text{ mm})(150\text{ mm})(150\text{ mm})$$
$$= 3{,}375{,}000\text{ mm}^3$$
$$\tau = \frac{VQ}{It}$$
$$= \frac{(39{,}200\text{ N})(3{,}375{,}000\text{ mm}^3)}{(1{,}139.1 \times 10^6\text{ mm}^4)(150\text{ mm})}$$
$$= \boxed{774\text{ kPa}}$$ **Ans.**

(b) Shear stress magnitude at *K*:

$$Q = (150\text{ mm})(100\text{ mm})(175\text{ mm})$$
$$= 2{,}625{,}000\text{ mm}^3$$
$$\tau = \frac{VQ}{It}$$
$$= \frac{(39{,}200\text{ N})(2{,}625{,}000\text{ mm}^3)}{(1{,}139.1 \times 10^6\text{ mm}^4)(150\text{ mm})}$$
$$= \boxed{602\text{ kPa}}$$ **Ans.**

(c) Maximum shear stress at any location:

$$Q_{max} = (150\text{ mm})(225\text{ mm})(112.5\text{ mm}) = 3{,}796{,}875\text{ mm}^3$$

$$\tau = \frac{VQ}{It} = \frac{(39{,}200\text{ N})(3{,}796{,}875\text{ mm}^3)}{(1{,}139.1 \times 10^6\text{ mm}^4)(150\text{ mm})} = \boxed{871\text{ kPa}}$$ **Ans.**

(d) Maximum bending stress at any location:

$$M_{max} = 39.2\text{ kN-m}$$

$$\sigma_x = -\frac{My}{I} = -\frac{(39.2\text{ kN-m})(225\text{ mm})(1{,}000\text{ N/kN})(1{,}000\text{ mm/m})}{1{,}139.1 \times 10^6\text{ mm}^4}$$
$$= -7.74296\text{ MPa} = \boxed{7{,}740\text{ kPa (C)}}$$ **Ans.**

- Shear and Bending Diagrams (https://bit.ly/2OodULo)
  - This video explains how to draw and derive the graphs of the shear and bending moment across the beam in question.

Example Problem:

Problem 5. (5 points) A simply supported beam supports the loads shown in Figure 5a. The cross-sectional dimensions of the structural tube shape are shown in Figure 5b

(a) Complete the shear force and bending moment diagrams

(b) At section a–a, which is located 7 ft to the left of roller support E, determine the bending stress and the shear stress at point H, which is located 3 in. below the top surface of the tube shape. Specify the sign of this bending stress.

(FBD required)

Figure 5a

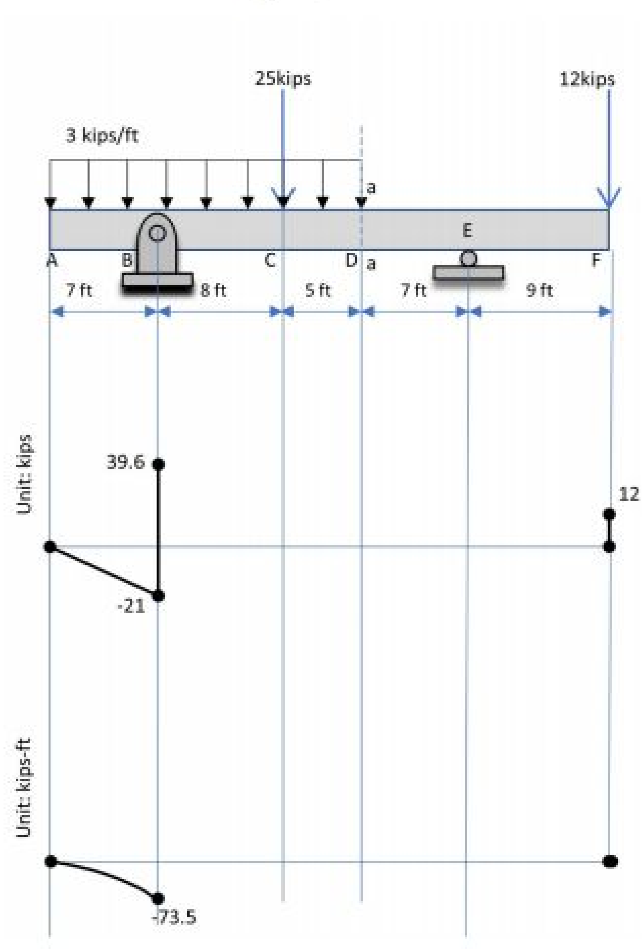
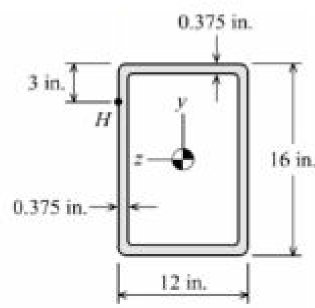


Figure 5b



No Solution Provided

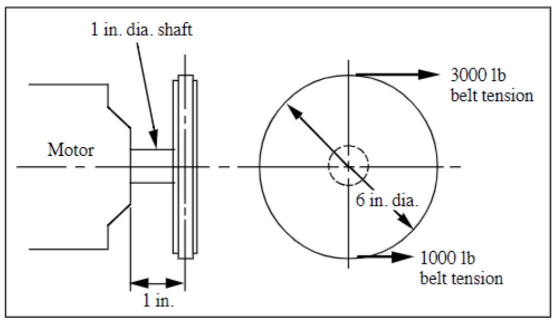
- ❖ Mohr's Circles and Principal Stresses (https://bit.ly/31NL2BS)
  - ➢ This video provides a simple explanation of how to solve for the principal normal and shear stresses and their angles using Mohr's Circle Analysis.

Example Problem with Solution:

Known: An electric motor is loaded by a belt drive.

Find: Copy the drawing and show on both views the location or locations on the shaft of the highest stress. Make a complete Mohr-circle representation of the stress at this location.

Schematic and Given Data:



Assumptions:
1. The weight of the structure is negligible.
2. The effect of stress concentration is negligible.
3. The shaft material is homogeneous and perfectly elastic.

Analysis:
1. For torsion, Eq. (4.4)

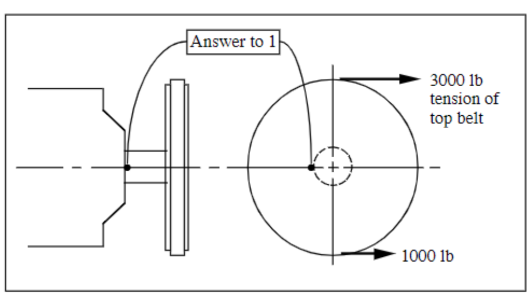
$$\tau = \frac{16T}{\pi d^3} = \frac{16(2000\text{ lb})(3\text{ in.})}{\pi(1\text{ in.})^3} = 31\text{ ksi}$$

2. For bending, Eq. (4.8)

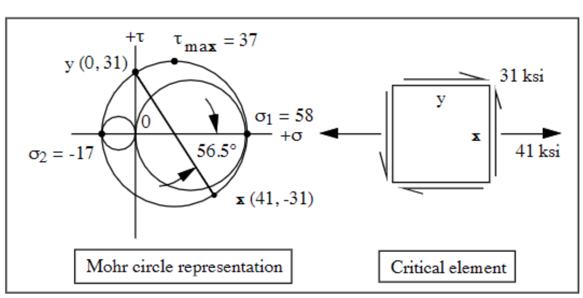
$$\sigma = \frac{32M}{\pi d^3} = \frac{32(4000\text{ lb})(1\text{ in.})}{\pi(1\text{ in.})^3} = 41\text{ ksi}$$

3. The Mohr circle representation is given above.

4.



5.



- ❖ Introduction to Finite Element Analysis (https://bit.ly/2VhxThS)
  - ➢ This video gives a non-software specific overview on the basics of F.E.A., putting into perspective how stress / strain are used conjunctively with computational methods to assess the risk of failure in a structural model.

As a general reference, here are some important formulas from MIE 211 that you may also find useful for the future:

Fundamental Mechanics of Materials Equations

Axial deformation

Deformation in axial members

$$\delta = \frac{FL}{AE} \quad \text{or} \quad \delta = \sum_i \frac{F_i L_i}{A_i E_i}$$

Force-temperature-deformation relationship

$$\delta = \frac{FL}{AE} + \alpha \Delta T L$$

Torsion

Maximum torsion shear stress in a circular shaft

$$\tau_{max} = \frac{Tc}{J}$$

where the polar moment of inertia J is defined as:

$$J = \frac{\pi}{2}[R^4 - r^4] = \frac{\pi}{32}[D^4 - d^4]$$

Gear relationships between gears A and B

$$\frac{T_A}{R_A} = \frac{T_B}{R_B} \qquad R_A\phi_A = -R_B\phi_B \qquad R_A\omega_A = R_B\omega_B$$

$$\text{Gear ratio} = \frac{R_B}{R_A} = \frac{D_B}{D_A} = \frac{N_B}{N_A}$$

Six rules for constructing shear-force and bending-moment diagrams

Rule 1: $\Delta V = P_0$

Rule 2: $\Delta V = V_2 - V_1 = \int_{x_1}^{x_2} w(x)\,dx$

Rule 3: $\frac{dV}{dx} = w(x)$

Rule 4: $\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V\,dx$

Rule 5: $\frac{dM}{dx} = V$

Rule 6: $\Delta M = -M_0$

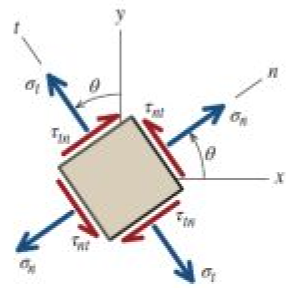
Flexure Formula

$$\sigma_x = -\frac{My}{I_z} \quad \text{or} \quad \sigma_{max} = \frac{Mc}{I} = \frac{M}{S} \quad \text{where } S = \frac{I}{c}$$

Horizontal shear stress associated with bending

$$\tau_H = \frac{VQ}{It} \qquad \text{where } Q = \Sigma\,\bar{y}_i A_i$$

Plane stress transformations



Stresses on an arbitrary plane

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$
$$\sigma_t = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta$$
$$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

Principal stress magnitudes

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Orientation of principal planes

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum in-plane shear stress magnitude

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{or} \quad \tau_{max} = \frac{\sigma_{p1} - \sigma_{p2}}{2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \qquad \text{note: } \theta_s = \theta_p \pm 45°$$

Absolute maximum shear stress magnitude

$$\tau_{abs\,max} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Normal stress invariance

$$\sigma_x + \sigma_y = \sigma_n + \sigma_t = \sigma_{p1} + \sigma_{p2}$$

Elastic curve relations between w, V, M, θ, and v for constant EI

Deflection $= v$

Slope $= \frac{dv}{dx} = \theta$ (for small deflections)